Advanced Automation and Control

June 21, 2019

Surname _____ Name _____

Part I - Optimization & Graphs (Prof. D.M. Raimondo)

1. Please solve the following MILP problem using the branch and bound algorithm

$$\begin{array}{ll} \max_{x_1,\delta_1,\delta_2} & 0.5x_1 + 2\delta_1 + 4\delta_2 \\ & -x_1 + \delta_2 \geq 3\delta_1 \\ & \delta_1, \delta_2 \in \{0,1\} \end{array}$$

2. Mr. John Doe is planning a few trips in the coming week. First, he has to go from Zürich to Milan. Then, once in Milan, he has to go to Lisbon. Finally, he will come back to Zürich.

   From Zürich to Milan (and from Milan to Zürich), he can take a train, use "bla-bla car" (a car-sharing service), or take his own car. The cost and duration for the different choices are reported in Table 1. Mr. Doe could take, for example, a train to Milan, and opt for another transport for the way back. However, in case he takes his own car to go to Milan, he will also go back to Zürich with his own car.

   When in Milan, Mr. Doe will go to Lisbon by plane. Once in Lisbon, he can decide to go to Milan, and then from Milan to Zürich, or to fly directly back to Zürich. The prices and duration of the different flights are reported in Table 2. Flying from Milan to Lisbon and from Lisbon to Zürich has a higher price than a return ticket from Milan to Lisbon and back, because two separate tickets have to be purchased.

   The objective of Mr. Doe is to minimise the cost given by the summation of two terms: first, the overall travel expenses, and second, a penalty which is accounted for only in case when the total travel time exceeds 12 hours. In this case, the penalty equals $20 \cdot (total\_travel\_time - 12)$.

   Please formulate Mr. Doe's situation as a mixed-integer linear programme.

Option	Cost	Duration
Train	19 €	4 h
Bla-bla car	25 €	3.3 h
His own car	35 €	3 h

Table 1: Cost and duration for the travel between Zürich and Milan and coming back.

Flight	Cost	Duration
Milan - Lisbon / Lisbon - Zürich	300 €	6.5 h
Milan - Lisbon / Lisbon - Milan	250 €	6 h

Table 2: Cost and duration for the flight.

3. Consider the following optimisation problem

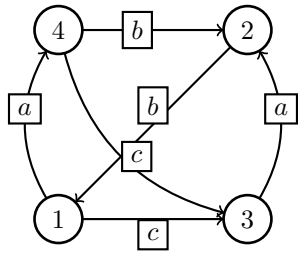
$$\begin{array}{ll} \min_{x_1,x_2} & 0.25x_1^2 + 9x_2^2 - 3x_1 \\ & x_1^2 + x_2^2 \leq 10 \\ & x_1^2 + x_2^2 \geq 3 \end{array}$$

   3.1 Indicate if the cost function is convex (motivate the answer).

   3.2 Indicate if the optimisation problem is convex (motivate the answer).

   3.3 Depict the feasibility domain of the problem.

4. Consider the automaton in the figure ($C = \{a, b, c\}$ is the set of control values and $S = \{1, 2, 3, 4\}$ is the set of state values) with the intermediate cost $g(x, u)$ and the terminal cost $g_3(x)$ given below



$g(x,u)$	a	b	c
1	1	-	3
2	-	5	-
3	3	-	-
4	-	4	2

$$g_3(x) = \begin{cases} 2 & \text{if } x = 1 \\ 1 & \text{if } x = 2 \\ 4 & \text{if } x = 3 \\ 4 & \text{if } x = 4 \end{cases}$$

4.1 Solve the optimal control problem

$$J(x_0) = \min_{u_0, u_1, u_2} g_3(x_2) + \sum_{k=0}^{2} g(x_k, u_k)$$

using dynamic programming.

4.2 Compute an optimal control sequence for $x_0 = 2$ and compute the optimal cost value.